Cambridge International AS & A Level

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MATHEMATICS **9709/41**

Paper 4 **May/June 2019**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Question	Answer	Mark	Guidance
1	$(X =)\ 78 \cdot\ 5/13 - 50 \times 3/5 = 78\cos 67.4 - 50\cos 53.1$ $(Y =)\ 78 \cdot\ 12/13 + 50 \times 4/5 - 112$ $= 78\sin 67.4 + 50\sin 53.1 - 112$	**M1**	Attempt to resolve forces either horizontally (2 terms) or vertically (3 terms)
	$[X = 30 - 30 = 0\ \ Y = 72 + 40 - 112 = 0]$	**A1**	Correct expressions horizontally and vertically
	$X = 0$ and $Y = 0$	**A1**	From convincing exact calculations
	Alternative method for question 1		
	$\frac{112}{\sin 59.5} = \frac{50}{\sin 157.4} = \frac{78}{\sin 143.1}$	**M1**	Attempt to use Lami, one pair of terms
		A1	All terms correct
	$\frac{112}{56/65} = \frac{50}{5/13} = \frac{78}{3/5} = 130$	**A1**	Exact values seen and used and shown to be = 130 $\cos[180 - (\theta + \alpha)] = 33/65$ and $\sin[180 - (\theta + \alpha)] = 56/65$
		3	

Question	Answer	Mark	Guidance
2(i)	$[0 = 25 - 10t]$	**M1**	Use of $v = u + at$ with $u = 25$, $v = 0$ and $a = -g$ or other complete method for finding t to highest point
	$t = 2.5$	**A1**	
		3	

Question	Answer	Mark	Guidance
2(ii)	[$20 = 25t - \frac{1}{2} gt^2$]	**M1**	Applying $s = ut + \frac{1}{2}at^2$ with $s = 20$, $u = 25$
	[$t = 1$ and $t = 4$]	**M1**	Solve a 3-term quadratic for t, factorising or formula
	Required time = 4 – 1 = 3 seconds	**A1**	
	Alternative method for question 2(ii)		
	[$v^2 = 25^2 + 2 \cdot (-10) \cdot 20 \quad \rightarrow \quad v = \pm 15$]	**M1**	Using $v^2 = u^2 + 2as$ with $u = 25$, $s = 20$ and $a = -g$
	[$-15 = 15 - 10T$] or equivalent	**M1**	Use v at $s = 20$ to find the time, T, taken to reach the maximum height and to return to $s = 20$
	Required time = 1.5 + 1.5 = 3 seconds	**A1**	
		3	
2(iii)	Max height reached at 2.5 s, hence reaches h after 2 s $h - 3 = 25 \cdot 2 - 5 \cdot 2^2$	**M1**	Using their t from 2(i) – 0.5 in $s = ut + \frac{1}{2}at^2$ Allow finding h without taking note of the additional 3 m
	$h = 33$ m	**A1**	
	Alternative method for question 2(iii)		
	Maximum height = $\frac{1}{2} \cdot (25 + 0) \cdot 2.5$ [= 31.25] o.e. In 0.5 s it falls distance $\frac{1}{2} \cdot 10 \times 0.5^2$ [= 1.25]	**M1**	For attempting to find both the maximum height and the distance fallen in 0.5 seconds
	$h = 31.25 - 1.25 + 3 = 33$ m	**A1**	
		2	

Question	Answer	Mark	Guidance
3(i)	DF = 1500 + 12 000 · g · 0.08 [DF = 11100]	**M1**	Using DF = Resistance + weight component (3 terms)
	Power = DF · 5	**M1**	Using $P = Fv$ (their 2 term DF · 5)
	Power = 11 100 · 5 = 55.5 kW	**A1**	**AG**
		3	
3(ii)	k · $5^2 = 1500$, $k = 60$	**B1**	**AG**
		1	
3(iii)	DF = $60v^2$	**B1**	Using DF = resistance = $60v^2$
	55500 = DF · $v = 60v^2$ · $v = 60v^3$	**M1**	$P = Fv$ used and attempt to solve a 2-term cubic equation for v
	$v = 9.74$ ms^{-1}	**A1**	
		3	

Question	Answer	Mark	Guidance
4(i)	$R = 13 \cos 67.4 = 13\ (5/13)$ [$R = 5$]	**B1**	Resolve forces perpendicular to plane. Allow 67.4 used
	$F + 13 \sin 67.4 = F + 13(12/13) = 20$ [$F = 8$]	**B1**	Resolve forces parallel to plane. Allow 67.4 used
		M1	Use $F = \mu R$
	$\mu = 8/5 = 1.6$	**A1**	**AG** Must be from exact working here
		4	

Question	Answer	Mark	Guidance
4(ii)	$13 \sin 67.4 - F = 1.3a$ $F = \mu R = 8 \quad \rightarrow \quad [4 = 1.3a]$	**M1**	For applying Newton's second law along the plane and also using $F = \mu R$ (3 terms)
	$a = 3.08$ ms^{-2}	**A1**	Allow $a = 40/13$
		2	
4(iii)	$s = 0 + 0.5 \cdot (40/13) \cdot 2^2$ [= 80/13 = 6.15]	**M1**	Use $s = ut + \frac{1}{2}at^2$ with $u = 0$ and their $a \neq \pm g$ to find the distance moved in the first 2 seconds
	WD = $8 \cdot 6.15$	**M1**	WD = $F \cdot d$
	WD = 49.2 J	**A1**	Allow WD = 640/13 J
	Alternative method for question 4(iii)		
	$s = 0 + 0.5 \cdot (40/13) \cdot 2^2$ [= 80/13 = 6.15]	**M1**	
	$[v = (40/13) \times 2]$ and $[\text{WD} = 1.3g(80/13)(12/13) - \frac{1}{2} \cdot 1.3 \cdot (80/13)^2]$	**M1**	Finding v after 2 seconds and using WD = PE loss – KE gain
	WD = 49.2 J	**A1**	Allow WD = 640/13 J
		3	

Question	Answer	Mark	Guidance
5(i)	$a = 2t - 8$	**M1**	Differentiate to find a
	$a = 0 \rightarrow t = 4$	**M1**	Set $a = 0$ and solve for t
	Minimum $v = -4$ ms^{-1}	**A1**	Full marks available for correct use of a v-t graph or correct use of "$t = -b/2a$"
	Alternative method for question 5(i)		
	$v = (t - 4)^2 - 4$	**M1**	Attempt to complete the square for v
	$[t = 4]$	**M1**	Choose the t value which gives minimum v
	Minimum $v = -4$ ms^{-1}	**A1**	
		3	
5(ii)	$v = 0$ when $(t - 2)(t - 6) = 0$	**M1**	Find values of t when $v = 0$, factorise or formula
	$t = 2$ or $t = 6$	**A1**	
	$[s = \frac{1}{3} t^3 - 4t^2 + 12t$ (+c)]	**M1**	Integrate v to find s
		A1	Correct integration
	$0 \leq t \leq 2$ $\quad s_1 = 8/3 - 16 + 24 = 32/3$ $2 \leq t \leq 6$ $s_2 = (216/3 - 144 + 72) - (8/3 - 16 + 24) = -32/3$ $6 \leq t \leq 8$ $s_3 = (512/3 - 4 \cdot 8^2 + 12 \cdot 8) - (216/3 - 144 + 72) = 32/3$	**M1**	Attempt to find s_1, s_2 and s_3 Look for consideration of the need for 3 intervals Allow use of symmetry when finding s_1, and s_3
		A1	2 correct values of displacement
	Total distance = 32 m	**A1**	All correct
		7	

Question	Answer	Mark	Guidance
6(i)	Particle A: $T = 4 \sin \theta$ Particle B: $T = 2$	**M1**	Resolve forces for A and for B
		M1	Eliminate T and solve for θ
	$\theta = 30$	**A1**	
		3	
6(ii)(a)	A: $T - 4 \sin 20 = 0.4a$ B: $2 - T = 0.2a$ System: $2 - 4 \sin 20 = (0.4 + 0.2)a$	**M1**	Apply Newton's second law to A or to B or to the system
		A1	Two correct equations
		M1	Solve for a or T
	$T = 1.79$ and $a = 1.05$	**A1**	Both correct
		4	
6(ii)(b)	$v^2 = 2 \cdot 1.053 \cdot 0.5 = 1.053$	**M1**	Attempt to find v using their $a \neq \pm g$
	$v = 1.03$ ms^{-1}	**A1**	
		2	

Question	Answer	Mark	Guidance
6(ii)(c)	Loss in KE = ½ · 0.4 · 1.053 = 0.2106 Gain in PE = 0.4 · 10 · d sin 20	**M1**	Attempt KE loss or PE gain for particle A only after particle B hits the ground.
		A1ft	Both correct, d is distance moved up the plane after B hits ground
	½ · 0.4 · 1.053 = 0.4 · 10 · d sin 20	**M1**	Apply KE loss = PE gain
		A1	**FT** Correct energy equation
	Total dist A moves up plane = 0.5 + d = 0.654 m	**A1**	
		5	